Discoveries and Vistas

Life, the universe, and higher categories

Celebrating the 60th birthday of Dominic Verity

Johns Hopkins University • Baltimore

July 20–22, 2026

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Discoveries and Vistas (DV60)

July 20–22, 2026

All talks in Krieger 205 · Johns Hopkins University, Baltimore

	Monday July 20	**Tuesday** July 21	**Wednesday** July 22
09:30	**Nick Gurski** Parity and flexibility	**Yuki Maehara** Towards a homotopy theory of algebraic weak ω-categories	**Mike Hopkins** The homotopy groups of spheres are purely algebraic
10:30	Coffee break		
11:00	**John Bourke** Accessibility of enriched and higher-dimensional categories	**Valeria de Paiva** (Zoom) Fibrational Semantics for Coexisting Implications	**Dom Verity** Discoveries and Vistas, a critical reflection
12:00	Lunch break		
14:30	**Steve Lack** Normal isofibrations	**Félix Loubaton** Triangles and globes	
15:30	Coffee break		
16:00	**Paula Verdugo** Towards real 2-Segal spaces	**Martina Rovelli** From (∞, n)-categories to (∞, ∞)-categories	
19:00		**Conference dinner** Glass Pavilion at Levering Hall	

Parity and flexibility

Nick Gurski

Joint work with: Niles Johnson

A Picard category is a symmetric monoidal groupoid in which every object has an inverse, up to isomorphism. These structures arise naturally in examples from algebra, geometry, and topology. A natural question to ask is: how does the presence of invertibility change the conclusions of the standard coherence theorem for symmetric monoidal categories? The first answer to this question was given by Dugger in [1] using a rewriting-style method. He proved that it is the parity of the morphisms, rather than their underlying symmetries, that determines if a given diagram commutes.

I have long championed an approach to this coherence theorem through the lens of 2-monad theory. Unfortunately, the "obvious" proof strategy had a gap where two different naturality conditions did not match. In the recent [2], we noticed that this incompatibility can be addressed by using the fact that the free Picard category generated by a single object is flexible as a symmetric monoidal category. In this talk, I will discuss our proof strategy, and if time permits preview ongoing work applying the same methods to braided monoidal structures.

[1] D. Dugger, *Coherence for invertible objects and multigraded homotopy rings*, Algebraic & Geometric Topology **14** (2014), 1055–1106.

[2] N. Gurski and N. Johnson, *Invertibility and parity in symmetric monoidal categories*, Applied Categorical Structures **34** (2026), article 34.

Accessibility of enriched and higher-dimensional categories

John Bourke

Joint work with: Steve Lack, Lukaš Vokřínek

In this talk, I will give an overview of joint research about accessibility and local presentability in higher dimensions, including 2-category theory and the infinity-cosmoi of Riehl and Verity.

The classical story is that accessible categories are those in which each object can built out of a set of generators in a natural way. They capture sets with infinitary first order structure. When they are complete, they are cocomplete, and vice-versa - in this setting, they are called locally presentable and capture algebraic structures of a general kind. One reason that such categories are important in practice is that they admit easily applicable adjoint functor theorems.

The aim of this project has been to study analogues of the above story in enriched category settings appropriate for higher category theory, and to obtain new useful results such as adjoint functor theorems of a homotopical flavour. It is much inspired by Australian 2-category theory [1, 2], Makkai's work on sketches [7], Lack and Rosický's work on enriched weakness [6] and Riehl and Verity's programme on studying infinity-categorical structures using 2-category theory and simplically-enriched category theory [8].

I will talk about some of the main ideas and insights, mainly from the papers [3, 4, 5] below, and also point to some recent developments and perhaps even some open questions.

[1] G. Bird, G.M. Kelly, A. J. Power and R. Street, *Flexible limits for 2-categories*, Journal of Pure and Applied Algebra 61,1989, 1–27.

[2] R. Blackwell, G.M. Kelly and A. J. Power, *Two-dimensional monad theory*, Journal of Pure and Applied Algebra 59, 1989, 1–41.

[3] John Bourke, *Accessible aspects of 2-category theory*, Journal of Pure and Applied Algebra 225(3), 2021, 106519.

[4] J. Bourke, S. Lack and L. Vokřínek. *Adjoint functor theorems for homotopically-enriched categories*, Advances in Mathematics 412, 2023, 108812.

[5] J. Bourke and S. Lack, *Accessible infinity-cosmoi*, Journal of Pure and Applied Algebra 227(5), 2023, 107255.

[6] S. Lack and J. Rosický, *Enriched weakness*, Journal of Pure and Applied Algebra 216, 2012, 1807–1822.

[7] M. Makkai, *Generalized sketches as a framework for completeness theorems. Part I*, Journal of Pure and Applied Algebra 115, 1997, 49–79.

[8] E. Riehl and D. Verity. *Elements of ∞-category theory*, Cambridge University Press, 2022.

Normal isofibrations

Steve Lack

Riehl and Verity [2] introduced the notion of ∞-cosmos in order to develop a "synthetic" theory of ∞-categories, analogous to the way in which a synthetic theory of ordinary categories can be developed in a 2-category. Every ∞-cosmos has an associated "homotopy" 2-category, and a surprising amount of the theory depends only on this 2-category.

The structure of an ∞-cosmos has three aspects: an enrichment over quasicategories, a class of maps called isofibrations, and certain (enriched) limits, including pullbacks of isofibrations along arbitrary maps.

The homotopy 2-category does not depend on the isofibrations (much as the homotopy category of a model category does not rely on the fibrations or cofibrations). Thus it is natural to wonder how much choice there might be in the isofibrations, and whether there is some sort of canonical choice.

John Bourke and I [1] studied this in the case of 2-categories, so that the enrichment is not just over quasicategories but over categories. We showed that (under very mild conditions) there is in fact a *minimal* choice of isofibrations, namely the *normal isofibrations*, and that any suitably complete 2-category becomes an ∞-cosmos when one takes the isofibrations to be these normal isofibrations.

What then happens if the enrichment is over quasicategories? It turns out that, under an assumption on the quasicategorically-enriched category that holds in all the main cases of interest, the notion of normal isofibration makes sense and there are analogues of the various results mentioned above. Indeed in most important examples of ∞-cosmoi, the chosen isofibrations are precisely the normal isofibrations. This forms part of the Master's project currently being completed by Declan Zammit.

In this talk I will talk about both the 2-categorical case and the general one.

[1] John Bourke and Stephen Lack. On 2-categorical ∞-cosmoi. *J. Pure Appl. Algebra*, 228(9):Paper No. 107661, 26, 2024.

[2] Emily Riehl and Dominic Verity. *Elements of ∞-category theory*, volume 194 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2022.

Towards real 2-Segal spaces

Paula Verdugo

Joint work with: Hadrian Heine

The notion of 2-Segal spaces, a generalization of that of 1-Segal spaces, encodes structures in which composition may not always exist or be unique. These structures were independently defined by Dyckerhoff and Kapranov in [2], and (by the name of decomposition spaces) by Gálvez-Carrillo, Kock, and Tonks in [3].

Both sets of authors identified the S-construction—a fundamental piece in algebraic K-theory—as a prominent source of examples of such structures. This connection was further studied by Bergner, Osorno, Ozornova, Rovelli, and Scheimbauer in [1], where they show that (a version of) the S-construction actually provides an equivalence between 2-Segal spaces and certain double Segal spaces.

In the realm of algebraic K-theory, we often care about extracting information in the presence of hermitian forms in our objects of study, which leads us to consider the so called *real* algebraic K-theory. In one of the existing approaches to this theory, we rely on a version of the S-construction introduced in [4]: the real S-construction.

Given the tight link in the classical case between the S-construction and 2-Segal spaces, a natural question arises: Is there a variant of 2-Segal spaces that is able to codify the extra structure on the real S-construction? In this talk, based on work in progress with Hadrian Heine, we discuss some obstacles for such generalization, and suggest a possible answer to this question.

[1] J. Bergner, A. Osorno, V. Ozornova, M. Rovelli, and C. Scheimbauer, *The S-construction as an equivalence between 2-Segal spaces and stable augmented double Segal spaces*, Contemp. Math., 2024.

[2] T. Dyckerhoff and M. Kapranov *Higher Segal Spaces*, Lecture Notes in Mathematics LNM, vol. 2244, Springer, 2019.

[3] I. Gálvez-Carrillo, J. Kock, A. Tonks, *Decomposition spaces, incidence algebras and Möbius inversion I: Basic theory*, Advances in Mathematics, vol. 331, 952-1015, Springer, 2018.

[4] H. Heine, M. Spitzweck, and P. Verdugo, *Real K-theory for Waldhausen ∞-categories with genuine duality*, arXiv:1911.11682, 2019.

Towards a homotopy theory of algebraic weak ω-categories

Yuki Maehara

Joint work with: Soichiro Fujii, Keisuke Hoshino

At one point during my PhD, my supervisor Dominic Verity told me that he thinks of complicial sets more as simplicial nerves of weak ω-categories than weak ω-categories themselves; if I remember correctly, one of the main reasons he gave was that the simplicial structure is amenable to only one kind of duality.

In that sense, it seems most intuitive to define weak ω-categories as some kind of globular sets. However, one cannot use horn-filling-like properties to encode the desired structure in a globular setting because globes do not have sufficiently many faces. This leads one to *algebraic* notions of weak ω-category.

In this talk, I will give an overview of our work [2, 3, 4] on algebraic weak ω-categories in the sense of Batanin [1] and Leinster [5], which are the Eilenberg-Moore algebras for a suitable monad (or, more accurately, a suitable globular operad) on the category of globular sets. Compared to more mainstream approaches (formulated in the language of model categories or ∞-categories), one downside of this definition is that it does not naturally come equipped with a homotopy theory. An upside, on the other hand, is that these algebraic weak ω-categories feel much closer to the *strict* ω-categories, particularly because not only the shapes of the cells but also the algebraic structure is designed to mirror the familiar behaviour of its strict counterpart. Therefore, the overarching theme of this project is to develop a suitable homotopy theory of the algebraic weak ω-categories by drawing as much intuition (and even proof strategies) as possible from the strict case.

[1] M. A. Batanin, *Monoidal globular categories as a natural environment for the theory of weak n-categories*, Adv. Math., 136(1):39–103, 1998.

[2] S. Fujii, K. Hoshino, and Y. Maehara, *Weakly invertible cells in a weak ω-category*, High. Struct., 8(2):386–415, 2024.

[3] S. Fujii, K. Hoshino, and Y. Maehara, *ω-weak equivalences between weak ω-categories*, Adv. Math., 480:110490, 2025.

[4] S. Fujii, K. Hoshino, and Y. Maehara, *ω-equifibrations between strict and weak ω-categories*, preprint arXiv:2511.09849, 2025.

[5] T. Leinster, *Higher operads, higher categories*, volume 298 of London Mathematical Society Lecture Note Series, Cambridge University Press, 2004.

Fibrational Semantics for Coexisting Implications

Valeria de Paiva

Joint work with: Milly Maietti, Eike Ritter

Most categorical semantics for systems combining intuitionistic and linear reasoning are organised around a modality or adjunction. Benton's LNL models and Barber–Plotkin's DILL models, for example, are built from symmetric monoidal adjunctions between a cartesian closed category and a symmetric monoidal closed category, with the modality (!) mediating between the intuitionistic and linear fragments. By contrast, the system we study here, TILL (Two-Implication Linear Lambda calculus, previously called ILT), is a non-modal double-context calculus in which both intuitionistic implication $(A \to B)$ and linear implication $(A \multimap B)$ are primitive, and the modality (!) is absent from the syntax. TILL is therefore not simply a presentation of DILL or LNL: it isolates the coexistence of intuitionistic and linear implication without any added modality structure.

We give a fibrational semantics for TILL by linearising the fibrational semantics of the simply typed (λ)-calculus. Concretely, we replace the cartesian fibres of a Jacobs-style comprehension category with symmetric monoidal closed fibres, while keeping the base category cartesian to model intuitionistic contexts. The resulting TILL-indexed categories provide a sound and complete semantics for TILL, and TILL is their internal language.

This fibrational perspective also explains how TILL sits in relation to the better-known modal systems. Modal structure can be recovered from TILL-indexed categories, yielding connections with the classical categorical semantics of DILL and LNL; in particular, DILL appears as a conservative extension of TILL. The core fibrational idea goes back to earlier joint work of the authors, but the full construction and proofs were never published. We present them here in full, thereby supplying a categorical semantics for TILL itself, rather than only for modal systems built around (!). In this sense, the paper identifies the non-modal semantic core of coexisting intuitionistic and linear implication, while preserving the modular structure that makes many ordinary (λ)-calculus techniques and implementations available on the intuitionistic side.

[1] M.E. Maietti, V. de Paiva, and E. Ritter, *Categorical models for intuitionistic and linear type theory*, International Conference on Foundations of Software Science and Computation Structures (2000), 223–237.

Triangles and globes

Félix Loubaton

When explaining higher categories to other mathematicians, one is sometimes tempted to make them believe that these are natural generalizations of categories, where one would allow cells of higher dimension. Obviously, this oversimplification elides the many choices and difficulties necessary to a working definition of higher categories, but one of them is particularly critical: the choice of the shape of these higher cells. Two candidates, however, stand out from the rest. The first, which we shall call globular, is arguably the closest to intuition, and the second, simplicial, allows one to connect higher categories to simplicial sets, these "lovely objects about which algebraic topologists know a lot", to quote Street. Identifying the right simplicial definition of higher categories, and the link it would have with their globular sibling, has been a rich program which has kept many mathematicians busy since the 70s and to which Dominic Verity has brought major contributions.

We will begin by recalling the definition of (globular) ω-categories, as well as the Roberts-Street Nerve $N : \omega\text{-cat} \to sSet$. We will use it to justify the definition of "complicial sets", and state the Roberts-Street Conjecture, which stipulates an equivalence between the globular and simplicial versions of higher categories, and which was resolved positively by Dominic Verity. From this, we will discuss the homotopical version of complicial sets, introduced by Street, and studied in depth by Verity. We will then state the homotopical version of the Roberts-Street Conjecture, resolved positively by the speaker, building on the work of Ozornova-Rovelli.

From (∞, n)-categories to (∞, ∞)-categories

Martina Rovelli

Joint work with: Viktoriya Ozornova, Tashi Walde

There is a well-developed theory of higher categories of finite dimension, in the form of strict n-categories in the strict setting and of (∞, n)-categories (for which n-complicial sets provide one model) in the weak setting. For higher categories of infinite dimension, however, the picture is subtler and less well understood. In the strict setting, a well-studied notion is that of an ω-category, while in the weak setting Verity introduced the notion of a complicial set.

In both contexts, the category of n-dimensional categories embeds into the corresponding category of infinite-dimensional categories. Moreover, this inclusion admits both a left and a right adjoint, yielding two natural ways to approximate an infinite-dimensional category by one of finite dimension. In this talk, we address the natural question of whether an infinite-dimensional category can be recovered from the family of its finite-dimensional approximations.

In the strict setting, an ω-category can be recovered both from its left truncations and from its right truncations. More precisely, the category of ω-categories is equivalent both to the limit of the tower of left truncations and to the limit of the tower of right truncations. The weak setting exhibits a markedly different behaviour. A complicial set can be recovered from its right truncations, but not in general from its left truncations. More precisely, the category of complicial sets is equivalent to the limit of the tower of right truncations [2, 4]. By contrast, the limit of the tower of left truncations embeds as a proper reflective subcategory of the category of complicial sets [1, 3].

[1] D. Gepner and H. Heine, *Homotopy Posets, Postnikov Towers, and Hypercompletions of ∞-Categories*, preprint, arXiv:2603.09903, 2026.

[2] F. Loubaton, *Theory and models of (∞, ω)-categories*, PhD thesis, preprint, arXiv:2307.11931, 2023.

[3] V. Ozornova, M. Rovelli and T. Walde, *Cores and localizations of (∞, ∞)-categories*, preprint arXiv:2603.11005, 2026

[4] V. Ozornova, M. Rovelli and T. Walde, *Recognition of limits for left and right towers*, in preparation, 2026.

Discoveries and Vistas, a critical reflection

Dom Verity

How pleasant to know Mr. Lear,
Who has written such volumes of stuff.
Some think him ill-tempered and queer,
But a few find him pleasant enough.
— Edward Lear (1871)

We'll take a critical tour through some of the dark nooks and crannies of my work. Along the way I hope to tease out some of the motivations that run through it and maybe even uncover a few mathematical landmines. Most importantly, I will signpost a few avenues left as yet unexplored, in the hope that they might motivate others to join me on this pilgrimage. I will also say a little about the type theoretic project currently frustrating me, and explain how it fits thematically in the landscape traversed here.